MR2231620 (2007d:54017) 54C65 (47H04 47H10 54H25)

Aksoy, A. G. (1-CMKC); **Khamsi, M. A.** (1-TXEP)

A selection theorem in metric trees. (English summary)

Proc. Amer. Math. Soc. **134** (2006), *no. 10,* 2957–2966 (*electronic*).

Let (M, d) be a metric tree, $\mathcal{C}$ denote the family of all nonempty bounded closed convex subsets of M and let d_H be the Hausdorff metric in $\mathcal{C}$. Among others the paper contains the following two results.

(a) If C is a nonempty convex subset of M, then for every $(x_\alpha)_{\alpha\in\Gamma}$ in M and $(r_\alpha)_{\alpha\in\Gamma}$ (r_α are positive numbers) such that $d(x_\alpha, x_\beta) \le r_\alpha + r_\beta$, $d(x_\alpha, C) \le r_\alpha$, $\alpha, \beta \in \Gamma$, the set $C \cap \bigcap_{\alpha\in\Gamma} B(x_\alpha, r_\alpha)$ is nonempty.

(b) If $T^*\colon M \to \mathcal{C}$, then there exists a selection T of T^* such that $d(T(x), T(y)) \le d_H(T^*(x), T^*(y))$ for all $x, y \in M$.

Reviewed by *W. Smajdor*

References

1. A. G. Aksoy, M. A. Khamsi, *Fixed points of uniformly Lipschitzian mappings in metric trees*. Preprint. cf. MR 2008f:54050
2. A. G. Aksoy, B. Maurizi, *Metric trees hyperconvex hulls and Extensions* . submitted.
3. N. Aronsza jn, P. Panitchpakdi, *Extension of uniformly continous transformations and hyperconvex metric spaces*. Pacific J. Math. 6 (1956), 405–439. MR0084762 (18,917c)
4. J. B. Baillon, *Nonexpansive mappings and hyperconvex spaces*. Contem. Math. 72 (1988), 11–19. MR0956475 (89k:54068)
5. I. Bartolini, P. Ciaccia, and M. Patella, *String matching with metric trees using approximate distance*. SPIR, Lecture Notes in Computer Science, Springer Verlag, Vol. 2476 (2002), 271–283.
6. Y. Benyamini, J. Lindenstrauss, *Geometric Nonlinear Functional Analysis*, AMS-Colloquium Publications, Vol. 48, 2000. MR1727673 (2001b:46001)
7. M. Bestvina, R-*trees in topology, geometry, and group theory*, Handbook of geometric topology, 55-91, North-Holland, Amsterdam, 2002. MR1886668 (2003b:20040)
8. L.M. Blumenthal, *Theory and applications of distance geometry*, Second Edition, Chelsea Publishing Co., New York, 1970. MR0268781 (42 #3678)
9. M. Bridson, A. Haefliger, *Metric spaces of nonpositive curvature*, Springer-Verlag, Berlin, Heidelberg, 1999. MR1744486 (2000k:53038)
10. P. Buneman, *A note on the metric properties of trees*. J. Combin. Theory Ser. B, 17 (1974), 48–50. MR0363963 (51 #218)
11. A. W. M. Dress, *Trees, tight extensions of metric spaces, and the chomological dimension of certain groups: a note on combinatorial properties of metric spaces*. Adv. in Math. 53 (1984), 321–402. MR0753872 (86j:05053)

12. A. W. M. Dress, V. Moulton and W. Terhalle, *T-Theory, An overview*. European J. Combin. 17 (1996), 161–175. MR1379369 (97e:05069)
13. J. R. Isbell, *Six theorems about injective metric spaces*. Comment. Math. Helv. 39 (1964), 439–447. MR0182949 (32 #431)
14. M. A. Khamsi, *KKM and Ky Fan Theorems in Hyperconvex Metric Spaces*. J. Math. Anal. Appl. 204 (1996), 298–306. MR1418536 (98h:54059)
15. M. A. Khamsi, and W. A. Kirk, "An Introduction to Metric Spaces and Fixed Point Theory". Pure and Applied Math., Wiley, New York, 2001. MR1818603 (2002b:46002)
16. M. A. Khamsi, W. A. Kirk, and C. Marinez-Ya nez, *Fixed points and selection theorems in hyperconvex spaces*. Proc. Amer. Math. Soc. 128, 11 (2000), 3275–3283. MR1777578 (2001g:54048)
17. M. A. Khamsi, M. Lin, and R. Sine, *On the fixed points of commuting nonexpansive maps in hyperconvex spaces*, J. Math. Anal. Appl. 168 (1992), 372–380. MR1175996 (93j:47080)
18. W. A. Kirk, *Hyperconvexity of* R-*trees*. Fund. Math. 156 (1998), 67–72. MR1610559 (98k:54060)
19. W. A. Kirk, Personal communication.
20. J. C. Mayer, L. K. Mohler, L. G. Oversteegen, and E. D. Tymchatyn, *Characterization of separable metric* R-*trees*. Proc. Amer. Math. Soc. 115, 1 (1992), 257–264. MR1124147 (92h:54049)
21. J. C. Mayer, L. G. Oversteegen, *A Topological Characterization of* R-*trees*. Trans. Amer. Math. Soc. 320, 1 (1990), 395–415. MR0961626 (90k:54031)
22. J. W. Morgan, **[U+2227]**-*trees and their applications*. **Bull. Amer. Math. Soc. 26 (1992), 87–112. MR1100579 (92e:20017)**
23. **F. Rimlinger,** *Free actions on* **R**-*trees*. **Trans. Amer. Math. Soc. 332 (1992), 313-329. MR1098433 (92j:20021)**
24. **C. Semple, and M. Steel,** *Phylogenetics*, **Oxford Lecture Series in Mathematics and its Applications, 24, 2003. MR2060009 (2005g:92024)**
25. **R. Sine,** *Hyperconvexity and nonexpansive multifunctions*. **Trans. Amer. Math. Soc. 315 (1989), 755–767. MR0954603 (90a:54054)**
26. **J. Tits,** *A Theorem of Lie-Kolchin for Trees*. **Contributions to algebra: a collection of papers dedicated to Ellis Kolchin, Academic Press, New York, 1977. MR0578488 (58 #28205)**
27. **M. Zippin,** *Application of Michael's continous selection theorem to operator extension problems*. **Proc. Amer. Math. Soc. 127, 5 (1999), 1371–1378. MR1487350 (99h:46040)**

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.